

Israel Open Astronomy Olympiad 2025

Junior and Senior age group set

Planetary system of Alphabet (Juniors: 30 p., Seniors: 50 p.)

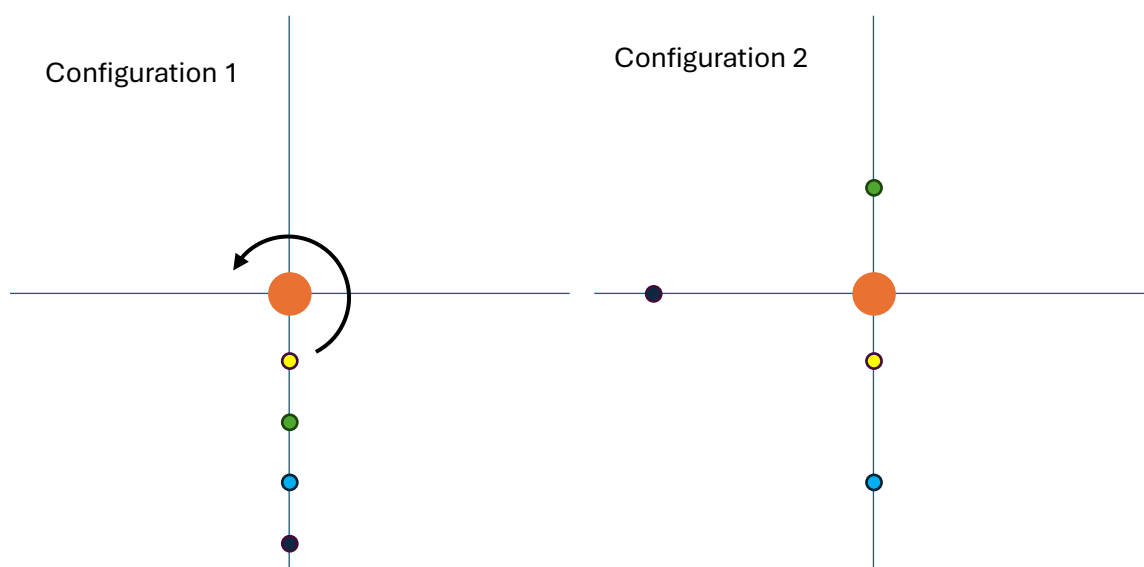
A planetary system exists around the star called Alphabet. The system contains four planets, which we will call A, B, C and D in the order from the central star outwards. The planets rotate around the star in one plane, in the same direction and on circular orbits. Two of the planets (B and C) are inhabited by a civilization that initially developed on the planet B.

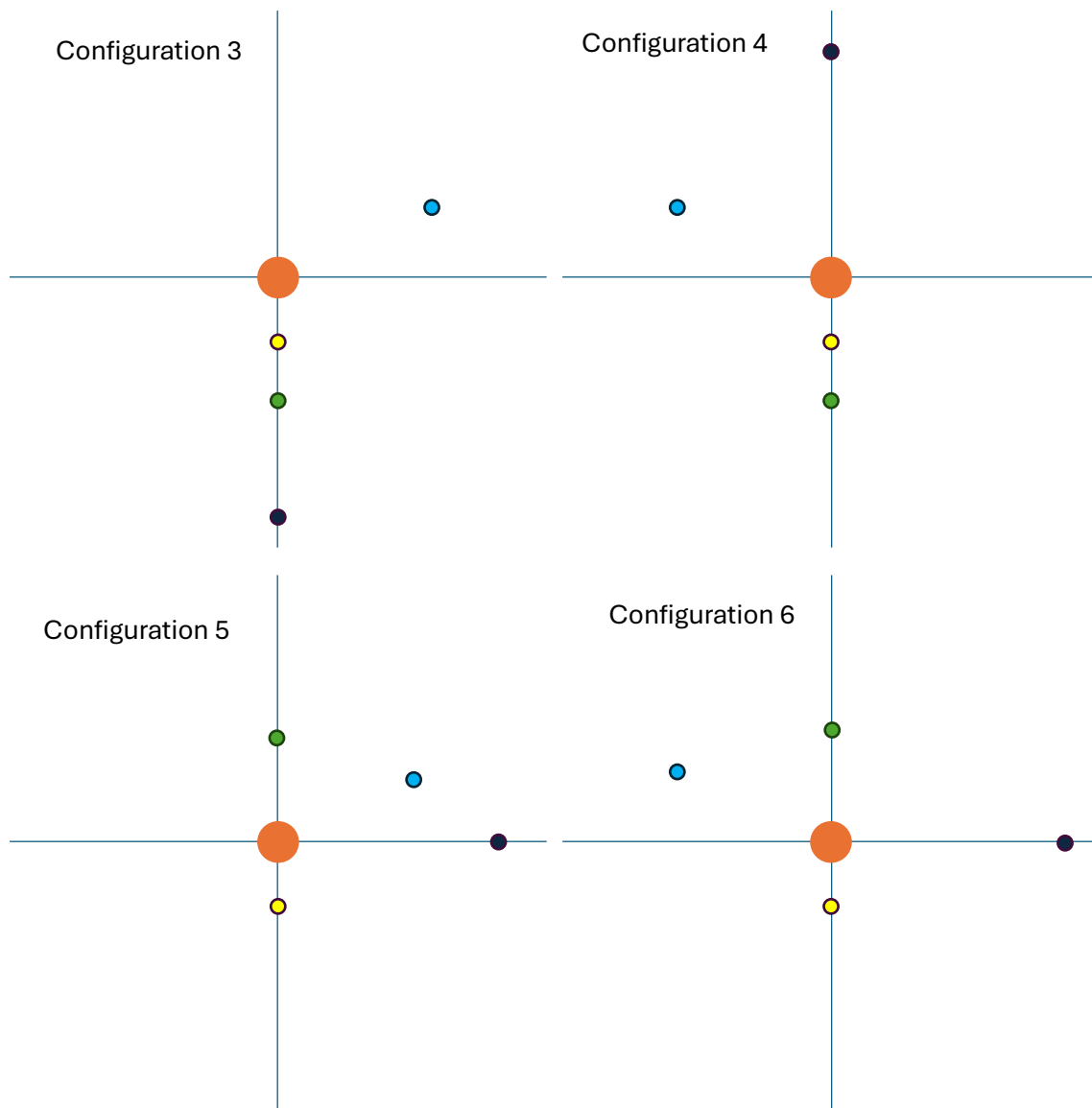
The ancient astronomers of planet B called the distance from the Alphabet to their planet B as *absolute unit* (au). By coincidence, 1 year of the planet B contains exactly 100 sols (solar days) of this planet.

A The images below show the planetary configurations as seen from the top, taken after each period of planet A. Orbiting direction of all planets is shown on the first image (Configuration 1) that corresponds to the initial time moment. Order the planetary configurations in time!

Answer: (2 p for each correct answer) The correct planetary configuration order is

- Configuration 1
- Configuration [2]/[3]/[4]/[5]/[6]
- Configuration [2]/[3]/[4]/[5]/[6]
- Configuration [2]/[3]/[4]/[5]/[6]
- Configuration [2]/[3]/[4]/[5]/[6]
- Configuration [2]/[3]/[4]/[5]/[6]



**Solution:**

In all configurations, the planet A is located below the Alphabet star. This is to be expected, since the configuration snapshots are taken after every orbit of planet A. The planet D is orbiting slower and may be seen in each of four cardinal directions. Therefore, planet D is making $\frac{1}{4}$ of the full orbit during one orbit of planet A. In the same way we conclude that during the same time the planet C makes $\frac{1}{3}$ of the orbit. It is enough to identify the order of the configurations.

The configuration next to "1" will have planet C pass $\frac{1}{3}$ of the circle and planet D pass $\frac{1}{4}$ of the circle. This is configuration "5". The next will have planet D pass $\frac{1}{2}$ of the circle; the only option is configuration "4". Following movement of the slowest planet D, we find that the next configurations are "2" (D on the left), "3" (D on the bottom) and "6" D again on the right.

Therefore, the correct order is 1 – 5 – 4 – 2 – 3 – 6.

B If the planets move around the star as shown on configuration diagrams above, then after how many years (orbital periods) of planet B the configurations of all planets will repeat? That is,

after how many years from Configuration 1 the planets will first be in Configuration 1 again?
(2 p.)

Answer: The Configuration 1 will repeat after each [] years of planet B.

Solution: The pairs A and B will be aligned again after each year of planet B; the pair B and C will be aligned after 3 years of planet B; the pair B and D will be aligned after 2 years of planet B. The minimum joint alignment will thus occur after 6 years of planet B.

C From planet B, all other planets are beautifully seen. The astronomers of planet B observed synodic periods of the inner planet A and outer planets C and D are shown in the table below. From this they computed their orbital (sidereal) periods in sols and orbital radii in au. The astronomers were also able to compute synodic periods of planets as seen from planet C. Repeat their calculations! *Note that the values you obtain here may differ from what you obtained in previous parts of the problem.*

Answer: Fill in the empty cells with the values that you compute (2 p. for every value).

	A planet	B planet	C planet	D planet
Synodic period as seen from B, sols	100	---	300	200
Orbital period, sols		1.0		
Orbit radius, au		1.0		
Synodic periods as seen from C, sols			---	

Solution:

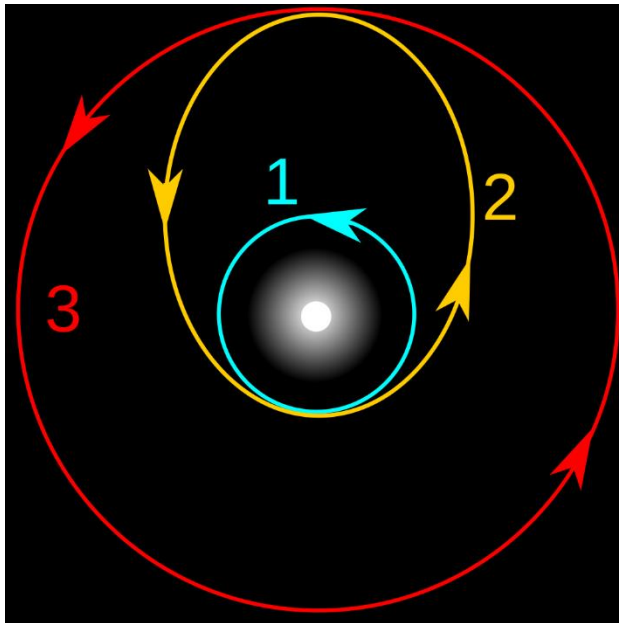
To determine the sidereal (orbital) period P , we will use the connection between the synodic S and sidereal periods. For inner planets, $\frac{1}{P} = \frac{1}{E} + \frac{1}{S}$ and for outer planets, $\frac{1}{P} = \frac{1}{E} - \frac{1}{S}$. Inserting the value of $E=100$ sol for orbital period of planet B, we obtain $P_A = 50$ sol, $P_C = 150$ sol, $P_D = 200$ sol. The planets are in an orbital resonance 1:2:3:4 similarly to the Galilean moon of Jupiter.

To obtain the orbit radius for each planet, we will apply the third Kepler's law, specifying that $T^2 = a^3$, if a is expressed in au, and T in planet B years. Inserting values, we obtain $a_A = 0,63$ au, $a_C = 1,31$ au, $a_D = 1,59$ au.

Looking from planet C, the synodic periods may be computed from sidereal ones. Inserting into the same expressions, we obtain $S_A = 75$ sol, $S_B = 300$ sol, $S_D = 600$ sol.

Note that synodic period of planet B, as seen from C, must be equal to synodic period of planet C, as seen from B, since this is the same period of repeating mutual planet configurations.

D How long should the spaceship fly from planet B to C? (5 p.) Assume that the radius of orbit of planet C is 1.5 au (*this may differ from the value you obtained previously*). The trajectory used by the spaceship is an ellipse touching in pericenter the orbit of planet B, and in apocenter touching the orbit of planet C. No engines are operated when moving on this trajectory.



The image shows elliptical Hohmann transfer orbit (2) used by the spaceship. It connects circular orbits of planet B (1) and planet C (3).

Answer: The flight time from B to C is [] sols

Solution:

The semimajor axis of the elliptic transfer orbit is 1.25 au (average of pericenter and apocenter distances), therefore the orbital period is $T = a^{3/2} = 1.397 \text{ y} = 139.7 \text{ sols}$. The time to reach planet C is half of the orbit, therefore the flight time is about 60 sol.

The next questions are only for Senior age group participants!

Astronomers of planet B were followers of decimal system and defined their time and distance units as factors of 10 and 100. According to them,

- The solar day (*sol*) contains 100 *minute* parts (*minute* means small), 1 sol = 100 min
- One *minute* part contains 100 *second* small parts, 1 min = 100 sec

Note that sols, minutes and seconds of planet B have no relation to our days, minutes and seconds.

E On the sky of planet B, its star Alphabet culminates exactly every one sol (which is also the definition of sol). How often does the Sun culminate in the sky of planet B? Remember that from planet B, the Sun is just one of many fixed stars.

Note: like Earth, the B planet rotates around its axis in the same direction as it orbits its star.

Answer: The Sun culminates every [] min. (3 p)

Solution: The Sun, being one of the fixed stars, culminates every sidereal day. As both rotation and orbital motion occur in the same direction, there are 100 sols and 101 sidereal days in one year of planet B. Therefore, 1 sidereal day is $100/101 \text{ sols} = 99 \text{ min } 1 \text{ sec}$. The answer 99 min is considered correct, 100 min is considered incorrect.

The scientists of planet B defined the distance unit, *moter*, in such a way that the planet B radius is 1 million *moters* = 1 000 000 m, or 1 000 *kilomoters* = 1000 km.

F The free fall acceleration on planet B is measured to be 100 m/sec^2 . Determine the circular velocity a spaceship must have to orbit the planet B on a low circular orbit! (5 p.) Determine the escape velocity a spaceship must have to escape the gravity of planet B! (2 p.) Express the answers in *kilomoters* per second, written as km/sec.

Answer: On planet B, the circular velocity is [] km/sec, and the escape velocity is [] km/s.

Solution:

The free fall acceleration g is derived from the Newton's law of gravity $mg = G \frac{Mm}{R^2}$ and therefore equals $g = \frac{GM}{R^2}$. Now we can use this value to determine the circular velocity: $v_c = \sqrt{\frac{GM}{R}} = \sqrt{gR}$.

Inserting the values, we get $v_c = \sqrt{100 \cdot 10^6} \text{ m/s} = 10 \text{ km/s}$.

The escape velocity $v_e = v_c \sqrt{2} = 14.1 \text{ km/s}$.

G To determine the connection between the *moter* m and the *absolute unit* au, the astronomers measured that the angular diameter of planet B, when observed from planet C, was equal to 1/100 000 of the full angle (that we on Earth call 360°), when the distance between the planets was 0.5 au. Determine, how many *moters* equals one *absolute unit*.

Answer: 1 au = [] m (5 p.)

Solution: If 2 radii of the planet, that is, $2 \cdot 10^6 \text{ moters}$, equals to 10^{-5} of the full angle, then the circumference of the circle with radius 0.5 au equals $2 \cdot 10^6 / 10^{-5} \text{ moters}$. That is,

$$2\pi \cdot 0.5 \text{ au} = 2 \cdot 10^{11} \text{ m}$$

Expressing from this the value of *absolute unit*, we get $1 \text{ au} = \frac{2}{\pi} \cdot 10^{11} \text{ m} = 6.37 \cdot 10^{10} \text{ m}$.

To measure the required angular diameter of planet B, it is not necessary to fly to planet C. It equals the so-called horizontal parallax of planet C and can be directly measured from planet B. A similar method was used for more than hundred years by Earth astronomers wishing to determine distance to the Sun in meters by observing horizontal parallax of Venus.

H Knowing that the distance from planet B to the star Alphabet equals 10^{10} moters (*this value may differ from the one received previously*), find the orbital speed of planet B in km/sec.

Answer: The orbital speed of planet B is [] km/sec. (5 p.)

Solution: The orbital speed v can be determined from the time T it takes to make one orbit, and the orbit circumference length L as

$$v = \frac{L}{T} = \frac{2\pi \cdot 10^{10} \text{ m}}{100 \cdot 100 \cdot 100 \text{ sec}} = 20\pi \frac{\text{km}}{\text{sec}} = 62.8 \frac{\text{km}}{\text{sec}}$$