Israel Open Astronomy Olympiad 2025

Senior age group problems

Eclipsing binary (60 p)

A binary star consists of two components orbiting around common center of mass. The parameters of the components are:

- Component A ("red") is a red giant, having mass $M_A = 4M_\odot$, radius $R_A = 30R_\odot$ and temperature $T_A = 3500$ K.
- Component B ("white") is a main sequence star with mass $M_A = 2M_\odot$, radius $R_A = 3R_\odot$ and temperature $T_A = 9500$ K.

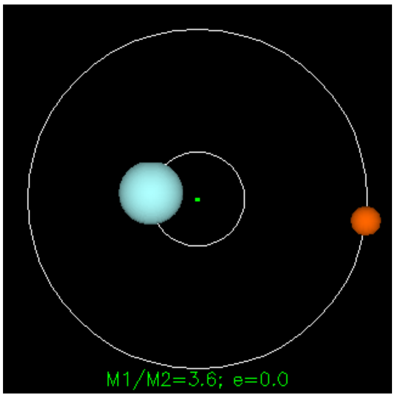


Figure 1. Binary stars orbiting common center of gravity.
Source: https://www.astronomy.ohio-state.edu/pogge.1/Ast162/Movies/vb0anim.gif

Part I. Magnitudes

A Determine the luminosity of each of the stars in the units of solar luminosity. (3 p for each value) Take required Solar values from the data sheet of the Olympiad.

Answer: Luminosity of component A is $L_A = [\qquad\qquad]L_\odot$, luminosity of component B is $L_B = [\qquad\qquad]L_\odot$.

Solution
The luminosity of the star according to the Stefan-Boltzmann law is proportional to the R^2T^4, where *R* and *T* are the radius and the temperature of the star. Therefore, the luminosity of the star $L_\star$ in units of solar luminosity is

$$\frac{L_\star}{L_\odot} = \frac{R_\star^2 T_\star^4}{R_\odot^2 T_\odot^4}$$

Inserting the values for the components, one obtains $L_A = \left(\frac{R_A}{R_\odot}\right)^2 \left(\frac{T_A}{T_\odot}\right)^4 L_\odot = 30^2 \left(\frac{3500}{5778}\right)^4 L_\odot = 121.17L_\odot$, and $L_B = 3^2 \cdot \left(\frac{9500}{5778}\right)^4 \cdot L_\odot = 65.77L_\odot$. The allowed range is 10%.

B Determine the absolute magnitude of each of the stars. Neglect the bolometric correction. (2 p for each value)

Answer: Absolute magnitude of component A is $M_A = [\qquad\qquad]^m$, absolute magnitude of component B is $M_B = [\qquad\qquad]^m$.

Solution
Pogson formula defines the difference of the apparent magnitudes based on difference in intensities. In the same way, the luminosity ratio is connected to the difference of absolute intensities:

$$\frac{L_\star}{L_\odot} = 10^{0.4(M_\odot - M_\star)}$$

Extracting the absolute magnitude of the star, we obtain that

$$M_\star = M_\odot - 2.5 \lg \frac{L_\star}{L_\odot}$$

Inserting the values, we obtain $M_A = +4.73 - 2.5 \lg 121.17 = -0.478^m$ and $M_B = +4.73 - 2.5 \lg 65.77 = +0.185^m$. The allowed range is 0.2^m.

C Determine the total absolute magnitude of the binary star. (1 p.)

Answer: Absolute magnitude of binary star is $M_{AB} = [\qquad\qquad]^m$.

Solution
To determine the total absolute magnitude, one must sum up the luminosities $L_{AB} = L_A + L_B$, and then determine the magnitude corresponding to the total luminosity as

$$M_{AB} = M_\odot - 2.5 \lg \frac{L_A + L_B}{L_\odot} = 4.73 - 2.5 \lg(121.17 + 65.77) = -0.949^m$$

D From the Earth, the binary star is measured to have the apparent magnitude $m_{AB} = +9.5^m$. Determine the distance to the binary star in parsec (3 p.). Neglect the interstellar absorption.
Answers: Distance to the binary star is $d = [\qquad\qquad]$ pc.

Solution
The absolute magnitude of the star equals to the apparent magnitude if the star is located at the standard distance of 10 pc. The apparent magnitude at any distance *d* (expressed in pc) is connected to the absolute magnitude via the intensities as seen at this distance

$$\frac{I(d)}{I(10\text{ pc})} = 10^{0.4(M-m)}$$

The intensity depends on distance according to the inverse square law as $I(d) = \frac{L}{4\pi d^2}$, therefore

$$\left(\frac{10\text{ pc}}{d}\right)^2 = 10^{0.4(M-m)}$$

Expressing the distance, we obtain

$$d = 10^{1+0.2(m-M)}$$

The equivalent expression $\lg d = 1 + 0.2(m - M)$ may be obtained from a well-known expression

$$m = M + 5 \lg d - 5$$

Inserting the values, we obtain $\lg d = 1 + 0.2 \cdot (9.5 + 0.949) = 3.090$ and $d = 1230$ pc.

E The parallax of the binary star is measured to be 0.0009'' with the uncertainty of 0.0001'' (that is, the parallax is measured to be in the range between 0.0008'' and 0.0010''). Determine the range of distances to the star in parsec from this measurement (1 p. for each value).

Answer: Based of the measured parallax value, the distance to the star is in the range from d_{min} = [ ] pc to d_{max} = [ ] pc.

Solution
The distance d and annual parallax *p* expressed in angular seconds are connected by relation $p = 1/r$. The range of the distances is then from $d_{min} = \frac{1}{p+\Delta p} = \frac{1}{0.0035} = 1000$ pc to $d_{max} = \frac{1}{p-\Delta p} = \frac{1}{0.0025} = 1250$ pc.

Part II. Eclipse magnitudes

The Solar system is located near the orbital plane of this binary star. This leads to events when (a) the smaller radius star is totally eclipsed ("eclipse") and (b) when the smaller star is transiting in front of the larger star ("transitions").

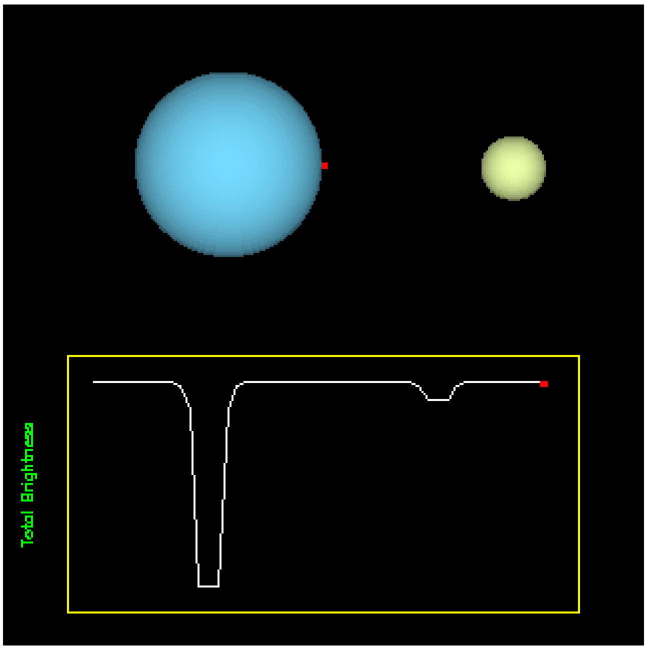


Figure 2. If we observe the binary star "from the side", the smaller component is periodically eclipsed by the larger, and periodically transits across the disk of the larger components. The lower diagram shows how the light curve relates to the star location on the orbit. Source: https://www.astronomy.ohio-state.edu/pogge.1/Ast162/Movies/eclbin.gif

In the next questions please **assume that distance to the binary star is 1490 pc, the absolute magnitude of the component A is -1^m and of component B is 0^m**. *These values may differ from the values obtained in previous questions, so don't use results of your earlier computations. You will have to repeat part of the computations.*

F Determine the apparent magnitude of the binary star during eclipse (2 p.) and during transition (10 p.) events!

Answer: Apparent magnitude of binary star during eclipse is $m_{ecl} = [\qquad]^m$ and during transition it is $m_{tr} = [\qquad]^m$.

Solution

During eclipse the smaller component B is fully eclipsed, and the intensity $I_{ecl} = I_A$ is determined only by the larger component A. The apparent magnitude is

$$m_{ecl} = M_A + 5\lg d - 5 = -1 + 5\lg 1490 - 5 = 9.866^m$$

For computation of the binary star magnitude during transit we need to know the ratio of the component intensities I_A/I_B. This can be obtained from Pogson's formula

$$\frac{I_A}{I_B} = 10^{0.4(M_B - M_A)} = 10^{0.4} = 2.512$$

During the transition of the smaller component by the disk of the larger component the observed intensity of component A is less than its total intensity by the covered part f of the disk of the larger component determined by radii of the components

$$f = \left(\frac{R_B}{R_A}\right)^2 = 0.01$$

The total intensity of the binary system during transit is then $I_{tr} = (1-f)I_A + I_B$. The corresponding absolute magnitude is determined from Pogson's formula as

$$M_{tr} = M_B - 2.5\lg\frac{I_{tr}}{I_B} = M_B - 2.5\lg\left[\left(\frac{I_A}{I_B}\right)\left(1 - \left(\frac{R_B}{R_A}\right)^2\right) + 1\right]$$

Inserting the values, we obtain $M_{tr} = 0^m - 2.5\lg[2.512 \cdot 0.99 + 1] = -1.356^m$

Finally, the apparent magnitude is $m_{tr} = M_{tr} + 5\lg d - 5 = -1.356 + 5\lg 1490 - 5 = 9.51^m$

G Look at the typical eclipsing binary star light curve. Please identify which phases of the light curve correspond to which reasons. (1 p. for each answer)

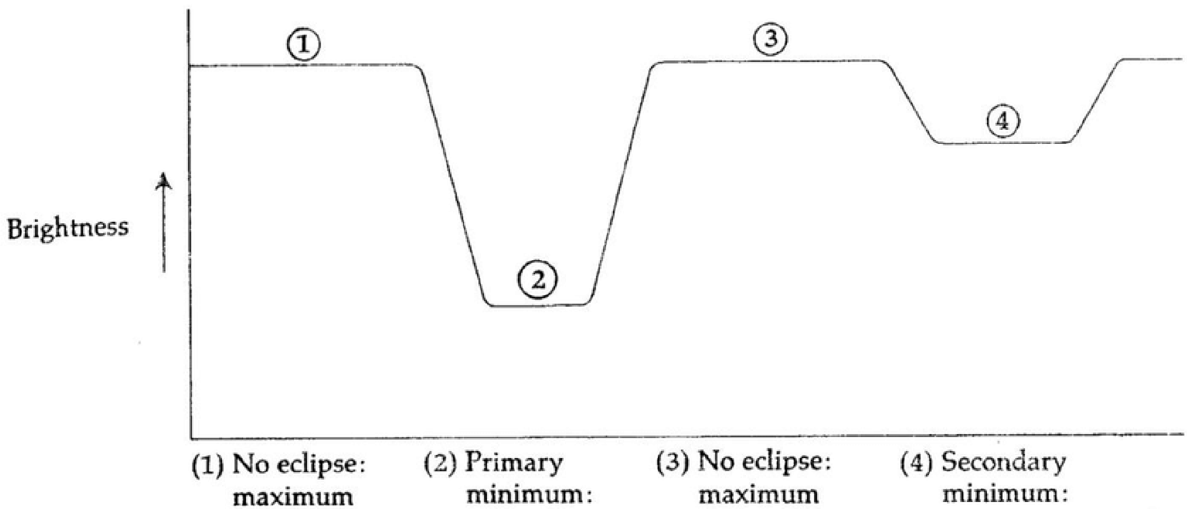
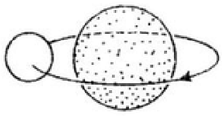
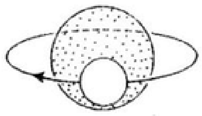


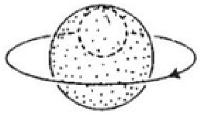
Figure 3. Light curve phases of the eclipsing binary star.



Case A: both stars are visible separately



Case B: the smaller components is transiting on the disk of the larger one



Case C: the smaller component is eclipsed by the larger one

Answer: Phase 1 on the light curve figure corresponds to the case [A]/[B]/[C], Phase 2 to [A]/[B]/[C], Phase 3 to [A]/[B]/[C], and phase 4 to [A]/[B]/[C].

Solution: As seen from previous estimates, the eclipse leads to much larger change in the apparent magnitude than the transition. Therefore, phases 1 and 3 correspond to case A, phase 2 to case C, phase 4 to case B.

Part III. Eclipse times

The period between the eclipses is $T_\star = 25$ days.

In this part we will utilize the concept of **relative orbit**, which is the orbit of one of the components if we assume that the second component is not moving. In the Figure below, the relative orbit is shown.

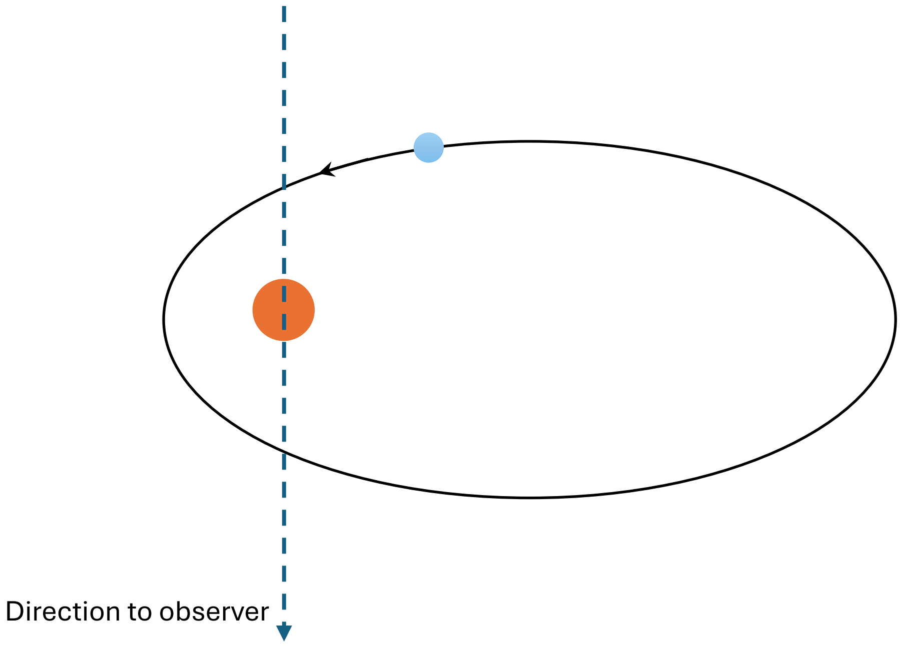


Figure 4. Relative orbit of the binary star and the direction to the observer.

H Determine the semimajor axis of the binary star relative orbit in au.

Astronomical unit or au equals to the value of the semimajor axis of the Earth orbit. In this question you may assume that the orbit is circular with the period as given above; then the semimajor axis is the same as the orbit radius. *Hint: compare movement of the component stars to movement of Earth around the Sun.*

Answer: Semimajor axis of the binary star relative orbit is $a_\star =$ [ ] au. (5 p)

Solution
The third Kepler's law with Newton's correction may be applied here comparing motion of Earth ⊕ around the Sun ⊙ and the binary star around the center of gravity.

$$\frac{(M_\odot + M_\oplus)T_\oplus^2}{a_\oplus^3} = \frac{(M_A + M_B)T_\star^2}{a_\star^3}$$

From this, we can express the semimajor axis as

$$a_\star = \left(\frac{25}{365.25}\right)^{\frac{2}{3}} \cdot \left(\frac{4M_\odot + 2M_\odot}{1M_\odot}\right)^{\frac{1}{3}} \cdot 1 = 0.304 \text{ au}$$

The orbit of the binary star is however not circular, but elliptic with the major axis perpendicular to the line of sight (see Figure 4 above). Eccentricity of the orbit equals $e = \frac{1}{\sqrt{2}} \approx 0.7071$.

In the final part of this problem, you should determine the time difference between the primary and secondary minimum events, that is, between the eclipse C and the transition D (see Figure below).

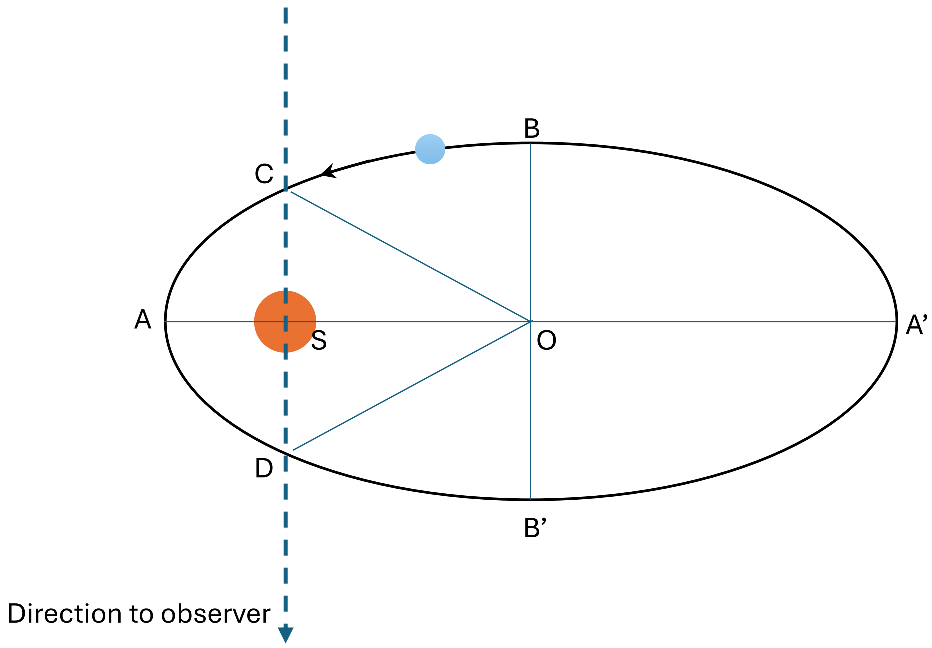


Figure 5. The relative orbit of the binary star.

This is a multistep process which is based on the second Kepler's law and geometrical properties of ellipse. The second Kepler's law states that for a binary star on a relative orbit "*the* *line connecting the stars sweeps equal areas in equal times"*, so we will have to compute the areas of parts of the ellipse.

To determine the fraction of the full period that passes between the eclipse and the transition, we will compare the area S_{ecl-tr} that the connecting line sweeps in this time t_{ecl-tr} with the full area of ellipse S_{ell}. That is, $t_{ecl-tr} = T \cdot \frac{S_{ecl-tr}}{S_{ell}}$.

The ellipse may be seen as a circle with radius $a = OA$ which is squeezed in one direction by factor b/a, where $b = OB$ is semiminor axis. The area of the ellipse therefore is $S_{ell} = \pi ab$.

I Which of the following figures encloses the area of the ellipse swept between the eclipse and the transition? (2 p.)

Answer: Select one of the options: [OCAS]/[OCAD]/[OBCADB']/[SCAD]

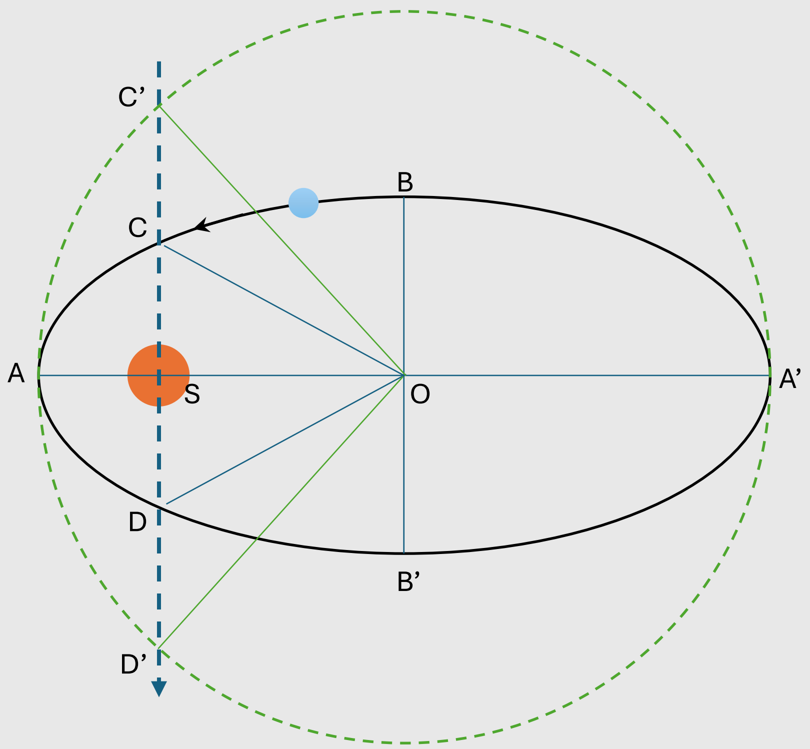
Solution
The line connecting the components starts at eclipse as SC and sweeps through SA until the transition that occurs at SD. The total enclosed area is thus SCAD.

Properties of the ellipse give the following relations: $OA = a$, $OB = b$, $OS = ae$, $SC = SD = b^2/a$. The eccentricity e may be expressed as $e = \sqrt{1 - b^2/a^2}$.

J Determine the area enclosed by the points OCAD.

Answer: $S_{OCAD} = ab \cdot [\qquad]$ (10 p.)

Solution
Let us determine first the area enclosed in part of a circle, and then squeeze it to ellipse.



Let us draw A circle with the same semimajor axis and the same center. We will continue the eclipse and transit line CD to the circle and obtain the points C'D'.

The distance $OS = ae = a/\sqrt{2}$, and the OC' is a radius of the circle of length a, therefore angle $\angle SOC' = \arccos\frac{1}{\sqrt{2}} = 45°$. Due to symmetry, $\angle SOD' = 45°$ and the $\angle C'OD' = 90°$. The area OC'AD' is then a quarter of the circle area, $\pi a^2/4$. Respectively, the area of the ellipse part OCAD is also a quarter of the ellipse area, $\pi ab/4$. Correct answer is $\pi/4 = 0.7854$.

K Determine the area enclosed by the points OCSD.

Answer: $S_{OCSD} = ab \cdot [\qquad]$ (5 p.)

Solution:

The area of the triangle OCD is $S_{OCD} = 2S_{OCS} = 2 \cdot \frac{1}{2} \cdot OS \cdot CS = ae \cdot \frac{b^2}{a} = eb^2 = \frac{1}{\sqrt{2}} \cdot \frac{ab}{\sqrt{2}} = \frac{1}{2}ab.$

The correct answer is therefore 0.5.

L Determine the area enclosed by the points CADS.

Answer: $S_{CADS} = ab \cdot [\qquad]$ (2 p.)

Solution

The remaining area is the difference $S_{CADS} = S_{OCAD} - S_{OCD} = ab \cdot \left(\frac{\pi}{4} - \frac{1}{2}\right) = 0.2854\ ab$. The correct answer is thus 0.2854.

M Determine the time from eclipse to transition events.

Answer: The time difference from eclipse event to transition event $t_{ecl-tr} = [\qquad]$ days. (4 p.)

Solution

According to the second Kepler's law, the time fraction equals to the area fraction, therefore

$$\frac{t_{ecl-tr}}{T_\star} = \frac{\left(\frac{\pi}{4} - \frac{1}{2}\right)\ ab}{\pi\ ab} = \frac{1}{4} - \frac{1}{2\pi} \approx 0.0908$$

Expressing in days, this is $t_{ecl-tr} = 0.0908 \cdot 25^d = 2.27^d$