Israel Open Astronomy Olympiad 2025

Senior age group problems

Time to ionize the Universe! (60 p)

Before the ultraviolet radiation from the early stars re-ionized the interstellar and intergalactic gas, it was completely neutral.

In this problem, we will consider the reionization of the early Universe by the first stars – so-called Population III stars. According to current models, the reionization was occurring roughly between the redshifts of 20 to 6.

To simplify the problem, let us use the following assumptions:

- The interstellar matter consists only of hydrogen atoms (note that hydrogen nucleus is just a proton, and hydrogen atom mass is roughly equal to proton mass);
- Most of stellar energy is produced when fusing hydrogen to helium;
- The ionization and recombination cross-sections do not depend on the radiation wavelength;
- The luminosity of a star is constant throughout its life.

Assume that the current average density of baryonic matter (protons, neutrons, electrons) of the Universe is $\rho_0 = 4.2 \cdot 10^{-28}$ kg/m^3. Use other constant values from the formula and constant page.

Let us consider a single Population III star that was born at a redshift of z = 19. The Universe was not yet so inhomogeneous at that time, and we will assume that outside the star forming region the matter density is still approximately constant and equals to the average matter density of the Universe.

A Compute the average baryonic matter density of the Universe at the time of the formation of this star. Express this in the mass density units (kg/m^3) as well as a number density of hydrogen (that is, average number of hydrogen atoms per m^3).

Answer: At redshift of z = 19, the mass density of baryonic matter is $\rho =$[   ] kg/m^3 (7 p.), that corresponds to the number density of hydrogen atoms $n_H =$[   ] m^{-3} (2 p.).

In all questions below, assume the number density of hydrogen atoms to be 1000 particles per cubic meter. This may differ from the result you obtained previously.

The Population III star that we consider had an initial mass of $M_0 = 80M_\odot$. Such stars are estimated to live only about $t = 3$ million years. During this time, they convert $p = 0.1$ of hydrogen they contain to helium.

B Compute the lifetime-average star luminosity (power of the emitted radiation) in W and in units of Solar luminosity $L_\odot$. When four protons combine to form a ^{4}He nucleus in the center of a star, about ε = 26.2 MeV energy is emitted and converted to radiation (1 MeV $= 1.6 \cdot 10^{-13}$ J). Neglect the energy required for initial heating of the star.

Answer: The lifetime-average luminosity of this star is [ ] W (10 p.) that converts to [ ] $L_{\odot}$ (2 p.).

In all questions below, assume that the star luminosity equals $L_{av} = 2 \cdot 10^{31}$ W. This may differ from the result you obtained previously.

About $P = 40\%$ of this luminosity is emitted in the energy range that ionizes neutral hydrogen, that is, above the so-called Lyman limit ($\varepsilon_{Ly} = 13.6$ eV). If such "ionizing" photon with any energy above Lyman limit is interacting with a neutral hydrogen atom, this atom gets ionized.

C Compute the rate at which the ionizing photons are emitted (that is, number of ionizing photons per second) by the star. Assume that the average energy of ionizing photons is $\varepsilon_{av} = 20$ eV.

Answer: This star emits approximately [ ] ionizing photons per second (6 p.).

In all questions below, assume that the ionizing photons are emitted with rate $L_{av} = 1 \cdot 10^{49}$ photons per second. This may differ from the result you obtained previously.

After this star starts to shine, its ionizing radiation creates an expanding sphere ("bubble") of ionized hydrogen, usually called a Strömgren sphere, located in an initially neutral surrounding interstellar medium. While the star is shining, this region gradually expands.

D Let us consider a Strömgren sphere with radius $R = 1$ kpc $= 1\ 000$ pc. Determine the ionization front speed, in km/s, that is, the speed, at which this ionized region around the star expands. Assume that the ionizing radiation is fully reaching the surface of the bubble and is fully absorbed there.

Here you should assume that the hydrogen is either fully ionized, or fully neutral, and that this ionized region has a sharp edge. Neglect the effect of the recombination.

Answer: The photoionized region around this star (the Strömgren sphere) expands at the speed [ ] km/s when its radius is 1 kpc = 1 000 pc (12 p.).

The protons do not interact with the ionizing radiation. However, they may recombine (join) with an electron, producing back a hydrogen atom. The rate of this process called recombination, that is, a probability that a single atom will recombine per time interval equals $r_{rec} = \alpha_{rec} \cdot n_e$, where n_e is the electron number density. The hydrogen recombination coefficient is $\alpha_{rec} = 3 \cdot 10^{-19}$ m³/s.

E Compute the amount of hydrogen recombinations per second in each cubic meter of fully ionized medium far from the star forming region $dN_{rec}/dt/dV$.

Answer: Due to recombination, there will be [ ] atoms formed from hydrogen ions every second in every cubic meter of ionized medium if the ionizing radiation will suddenly disappear (7 p.).

In all questions below, assume that the requested value of the hydrogen recombinations per unit volume and unit time equals $\frac{dN_{rec}}{dt \cdot dV} = 1 \cdot 10^{-12}\,\frac{1}{\mathrm{s\,m^3}}$. *This may differ from the result you obtained previously.*

F Determine the equilibrium volume and equilibrium radius of the Strömgren sphere V_S and R_S, when the number of the ionizing photons will just be enough to keep the recombining ions in the sphere ionized. Express the answer in kpc^3 and kpc (kiloparsec), respectively.

Answer: The Strömgren sphere will grow until reaching the equilibrium volume of [ ] kpc^3 (10 p.) corresponding to equilibrium radius of [ ] kpc (4 p.) and then stop growing due to hydrogen recombination occurring inside it.