Israel Open Astronomy Olympiad 2025

Senior age group problems

Time to ionize the Universe! (60 p)

Before the ultraviolet radiation from the early stars re-ionized the interstellar and intergalactic gas, it was completely neutral.

In this problem, we will consider the reionization of the early Universe by the first stars – so-called Population III stars. According to current models, the reionization was occurring roughly between the redshifts of 20 to 6.

To simplify the problem, let us use the following assumptions:

- The interstellar matter consists only of hydrogen atoms (note that hydrogen nucleus is just a proton, and hydrogen atom mass is roughly equal to proton mass);
- Most of stellar energy is produced when fusing hydrogen to helium;
- The ionization and recombination cross-sections do not depend on the radiation wavelength;
- The luminosity of a star is constant throughout its life.

Assume that the current average density of baryonic matter (protons, neutrons, electrons) of the Universe is $\rho_0 = 4.2 \cdot 10^{-28}$ kg/m^3. Use other constant values from the formula and constant page.

Let us consider a single Population III star that was born at a redshift of z = 19. The Universe was not yet so inhomogeneous at that time, and we will assume that outside the star forming region the matter density is still approximately constant and equals to the average matter density of the Universe.

A Compute the average baryonic matter density of the Universe at the time of the formation of this star. Express this in the mass density units (kg/m^3) as well as a number density of hydrogen (that is, average number of hydrogen atoms per m^3).

Answer: At redshift of z = 19, the mass density of baryonic matter is $\rho =$[ ] kg/m^3 (7 p.), that corresponds to the number density of hydrogen atoms $n_H =$[ ] m^{-3} (2 p.).

Solution:

The scale factor of the Universe at the considered time was equal to $a = \frac{1}{z+1} = \frac{1}{20}$, meaning that the Universe was 20 times smaller in every direction (this does <u>not</u> imply that the Universe is finite). The matter density was then $a^3 = 8000$ times larger than it is now, that is, $\rho(z) = (z+1)^3 \rho_0$ and equal to $3.36 \cdot 10^{-24}$ kg/m^3.

The hydrogen atom mass is roughly equal to the proton mass $m_p = 1.673 \cdot 10^{-27}$ kg. The number density of atoms was then $n_H(z) = \frac{\rho(z)}{m_p} = \frac{3.36 \cdot 10^{-24}}{1.673 \cdot 10^{-25}} = 2\ 010$ particles per m^3.

At that time there was about two particles per liter on average in the Universe, and this is still 8000 times more than what we have now. The Universe is extremely rarefied on average and stars and planets are result of long gravitational collapse of some part of this matter.

In all questions below, assume the number density of hydrogen atoms to be 1000 particles per cubic meter. This may differ from the result you obtained previously.

The Population III star that we consider had an initial mass of $M_0 = 80 M_\odot$. Such stars are estimated to live only about $t = 3$ million years. During this time, they convert $p = 0.1$ of hydrogen they contain to helium.

B Compute the lifetime-average star luminosity (power of the emitted radiation) in W and in units of Solar luminosity $L_\odot$. When four protons combine to form a ^{4}He nucleus in the center of a star, about ε = 26.2 MeV energy is emitted and converted to radiation (1 MeV = $1.6 \cdot 10^{-13}$ J). Neglect the energy required for initial heating of the star.

Answer: The lifetime-average luminosity of this star is [ ] W (10 p.) that converts to [ ] $L_\odot$ (2 p.).

Solution: The energy produced in converting four protons (having mass $4m_p = 6.692 \cdot 10^{-27}$ kg) into helium is $\varepsilon = 26.2 \text{ MeV} = 4.20 \cdot 10^{-12}$ J, therefore the total energy emitted in the lifetime of the star will be

$$E_{tot} = pM_0 \cdot \frac{\varepsilon}{4m_p}$$

So the average luminosity is

$$L_{av} = \frac{E_{tot}}{t} = \frac{pM_0}{t} \cdot \frac{\varepsilon}{4m_p}$$

Inserting the values, we obtain $L_{av} = 1.06 \cdot 10^{32}$ W or $2.77 \cdot 10^5$ $L_\odot$. This luminosity is much larger than that of the Sun, but below the brightest known stars (mostly, Wolf-Rayet stars and so-called Luminous blue variable stars) in the Milky Way, and similar e.g. to Alnitak, one of the stars in the Orion belt and a bright O-type star.

In all questions below, assume that the star luminosity equals $L_{av} = 2 \cdot 10^{31}$ W. This may differ from the result you obtained previously.

About $P = 40\%$ of this luminosity is emitted in the energy range that ionizes neutral hydrogen, that is, above the so-called Lyman limit ($\varepsilon_{Ly} = 13.6$ eV). If such "ionizing" photon with any energy above Lyman limit is interacting with a neutral hydrogen atom, this atom gets ionized.

C Compute the rate at which the ionizing photons are emitted (that is, number of ionizing photons per second) by the star. Assume that the average energy of ionizing photons is $\varepsilon_{av} =$ 20 eV.

Answer: This star emits approximately [ ] ionizing photons per second (6 p.).

Solution:

The number of ionizing photons emitted per second is the ionizing part of the luminosity of the star $P \cdot L_{av}$ divided by average amount of energy each photon contains $\varepsilon_{av} = 20\ \text{eV} = 3.20 \cdot 10^{-18}$ J. Numerically, $\frac{dN_{ionizing}}{dt} = \frac{P \cdot L_{av}}{\varepsilon_{av}} = 2.50 \cdot 10^{48}$ ionizing photons per second.

In all questions below, assume that the ionizing photons are emitted with rate $L_{av} = 1 \cdot 10^{49}$ photons per second. This may differ from the result you obtained previously.

After this star starts to shine, its ionizing radiation creates an expanding sphere ("bubble") of ionized hydrogen, usually called a Strömgren sphere, located in an initially neutral surrounding interstellar medium. While the star is shining, this region gradually expands.

D Let us consider a Strömgren sphere with radius $R = 1\ \text{kpc} = 1\ 000\ \text{pc}$. Determine the ionization front speed, in km/s, that is, the speed, at which this ionized region around the star expands. Assume that the ionizing radiation is fully reaching the surface of the bubble and is fully absorbed there.

Here you should assume that the hydrogen is either fully ionized, or fully neutral, and that this ionized region has a sharp edge. Neglect the effect of the recombination.

Answer: The photoionized region around this star (the Strömgren sphere) expands at the speed [ ] km/s when its radius is 1 kpc = 1 000 pc (12 p.).

Solution:

The ionization front speed is determined by the equality of number of ionizing photons to the number of ionized atoms $\frac{dN_{ionizing}}{dt} = \frac{dN_{ionized}}{dt}$. The emission rate of ionizing photons was computed in the previous question and equals $\frac{dNionizing}{dt} = 1.33 \cdot 10^{49}$ photons/second. The number of ionized atoms, if the ionization front passes distance ds, is $dN_{ionized} = dV \cdot n_H = 4\pi R^2 \cdot ds \cdot n_H$. The factor $4\pi R^2 \cdot n_H = 4\pi \cdot (10^3 \cdot 3.086 \cdot 10^{16}\ \text{m})^2 \cdot 2.01 \cdot 10^3\ \text{m}^{-3} = 2.41 \cdot 10^{43}$ m^{-1}, meaning that increasing the ionized region by one meter requires $2.41 \cdot 10^{43}$ ionizing photons.

Expressing the ionization front speed $v = ds/dt$, we obtain

$$v = \frac{ds}{dt} = \frac{P \cdot L_{av}}{\varepsilon_{av}} \cdot \frac{1}{4\pi R^2 n_H} = \frac{1.33 \cdot 10^{49}\ \text{s}^{-1}}{2.41 \cdot 10^{43}\ m^{-1}} = 2.09 \cdot 10^5 \frac{m}{s} = 209\ \text{km/s}$$

Note that 1000 pc = 1 kpc is about 10% of the distance from the Sun to the center of the Milky way and is comparable to the size of modern dwarf galaxies. So even a single Population III star was able to ionize the interstellar medium around its formation region. Based on computer simulations, it is expected that the Population III stars were born in groups, that were rapidly dispersing the remaining gas in their forming region and ionizing huge expanses of space around them.

The protons do not interact with the ionizing radiation. However, they may recombine (join) with an electron, producing back a hydrogen atom. The rate of this process called recombination, that is, a probability that a single atom will recombine per time interval equals $r_{rec} = \alpha_{rec} \cdot n_e$, where n_e is the electron number density. The hydrogen recombination coefficient is $\alpha_{rec} = 3 \cdot 10^{-19}$ m^3/s.

E Compute the amount of hydrogen recombinations per second in each cubic meter of fully ionized medium far from the star forming region $dN_{rec}/dt/dV$.

Answer: Due to recombination, there will be [ ] atoms formed from hydrogen ions every second in every cubic meter of ionized medium if the ionizing radiation will suddenly disappear (7 p.).

Solution:

As we assume that the primordial plasma consists purely of hydrogen, the number density of electrons in the fully ionized medium equals the number density of protons. Therefore the recombination rate per unit volume will equal

$$\frac{dN_{rec}}{dt \cdot dV} = r_{rec} \cdot n_H = \alpha_{rec} n_e n_H = \alpha_{rec} n_H^2$$

Numerically, there will be $3.00 \cdot 10^{-13}$ recombination events per second per m^3.

It is not asked in the question, but this implies the fully ionized hydrogen will recombine after time $t_{rec} = \frac{1}{\alpha_{rec} n_H} \approx \frac{2000}{3.00 \cdot 10^{-13}} = 211$ million years, comparable to the age of the universe at $z = 19$ which was also about 200 million years.

As the universe expands, the matter density drops and the recombination time grows. Currently, when the matter number density is 8000 times less, the recombination time of fully ionized plasma far from galaxies is about 8000 times longer, that is, much longer than the age of the universe. One may conclude that the intergalactic medium once ionized by the first stars, will never recombine due to the continuing expansion of the universe, even if no ionizing radiation will be emitted.

In all questions below, assume that the requested value of the hydrogen recombinations per unit volume and unit time equals $\frac{dN_{rec}}{dt \cdot dV} = 1 \cdot 10^{-12} \frac{1}{\mathrm{s\,m^3}}$*. This may differ from the result you obtained previously.*

F Determine the equilibrium volume and equilibrium radius of the Strömgren sphere V_S and R_S, when the number of the ionizing photons will just be enough to keep the recombining ions in the sphere ionized. Express the answer in kpc^3 and kpc (kiloparsec), respectively.

Answer: The Strömgren sphere will grow until reaching the equilibrium volume of [ ] kpc^3 (10 p.) corresponding to equilibrium radius of [ ] kpc (4 p.) and then stop growing due to hydrogen recombination occurring inside it.

Solution:

The Strömgren sphere radius will be defined by the equilibrium between ionization and recombination events: $\frac{dN_{ionizing}}{dt} = \frac{dN_{rec}}{dt}$. The number of recombination events in the ionized volume is $\frac{dN_{rec}}{dt} = \frac{dN_{rec}}{dt\, dV} \cdot V$, where $V = \frac{4}{3}\pi R^3$ is the volume of the Strömgren sphere. Expressing the radius from the equations above, we obtain its equilibrium value:

$$R_{eq} = \sqrt[3]{\frac{3V_{eq}}{4\pi}} = \sqrt[3]{\frac{3}{4\pi} \cdot \frac{\frac{dN_{ionizing}}{dt}}{\frac{dN_{rec}}{dt\,dV}}}$$

Numerically, the equilibrium volume is $V_{eq} = 1.0 \cdot 10^{61}\ \mathrm{m}^3 = 340\ \mathrm{kpc}^3$. The equilibrium radius of the ionized sphere is then $R_{eq} = 4.33$ kpc.

A single Population III star thus was able to ionize the volume similar to the one of the Large Magellanic Cloud.

The real ionized regions around the first stars were not spherical, since the distribution of matter was not symmetric. Simulations of the reionization of the Universe show patchy structure of expanding regions that start to merge at some point. A notable example is a Thesan simulation, available at www.thesan-project.com, that computed e.g. shapes and movement of large-scale ionization fronts around clusters of early stars.