Israel Open Astronomy Olympiad 2025

Senior age group problems

Distance to M100 (90 p)

One of major problems in extragalactic astronomy of 20th century was establishing a distance scale outside the Milky Way. The problem of determining exact distances still stands open, as demonstrated by the ongoing Hubble tension, but the scale of the tension is much smaller.

Historically the first such method used observations of Cepheids. The Cepheids are very bright pulsating variable stars, named after their "prototype star" δ Cephei. Already more than 100 years ago Henriette Leavitt discovered that absolute magnitude of these stars relates to their pulsation period. This allowed Edwin Hubble to determine the distance to the Andromeda nebula and prove that this nebula (now known as Andromeda galaxy) is situated far outside the Milky Way galaxy. Also now, the Cepheid method is one of the leading methods to determine distances in the nearby Universe.

In this problem, you will use this method to determine distance to Messier 100 (in short, M100) galaxy belonging to the Virgo cluster. Two images of M100 are shown below.

Figure 1. Spiral galaxy M100 (on the lower left) as photographed by an amateur telescope. Source: http://madhuprathi.com/m100/

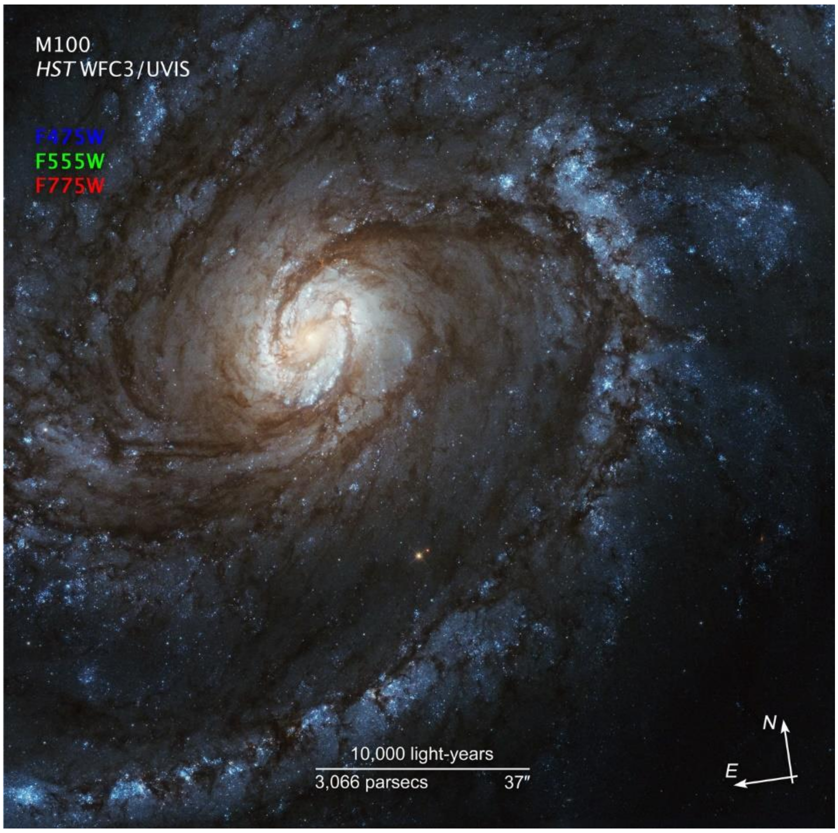


Figure 2. Spiral galaxy M100 as observed by Hubble Space Telescope. Source: hubblesite.org

Hubble Space Telescope has discovered and measured light curves of many Cepheids in the M100 galaxy. An example of how individual Cepheid images look like is shown on the Figure below.

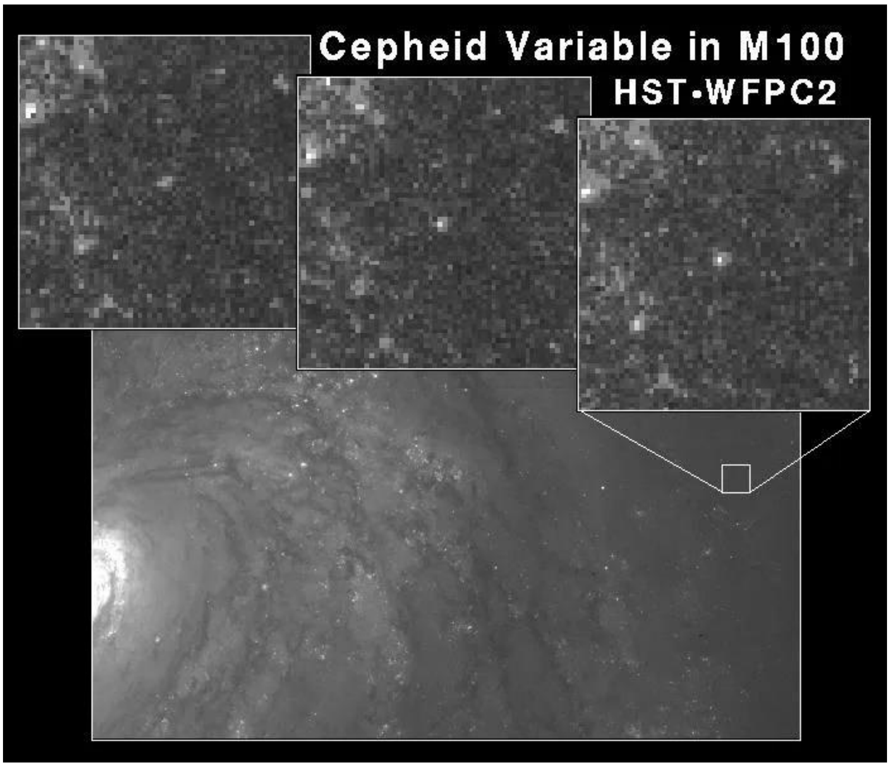
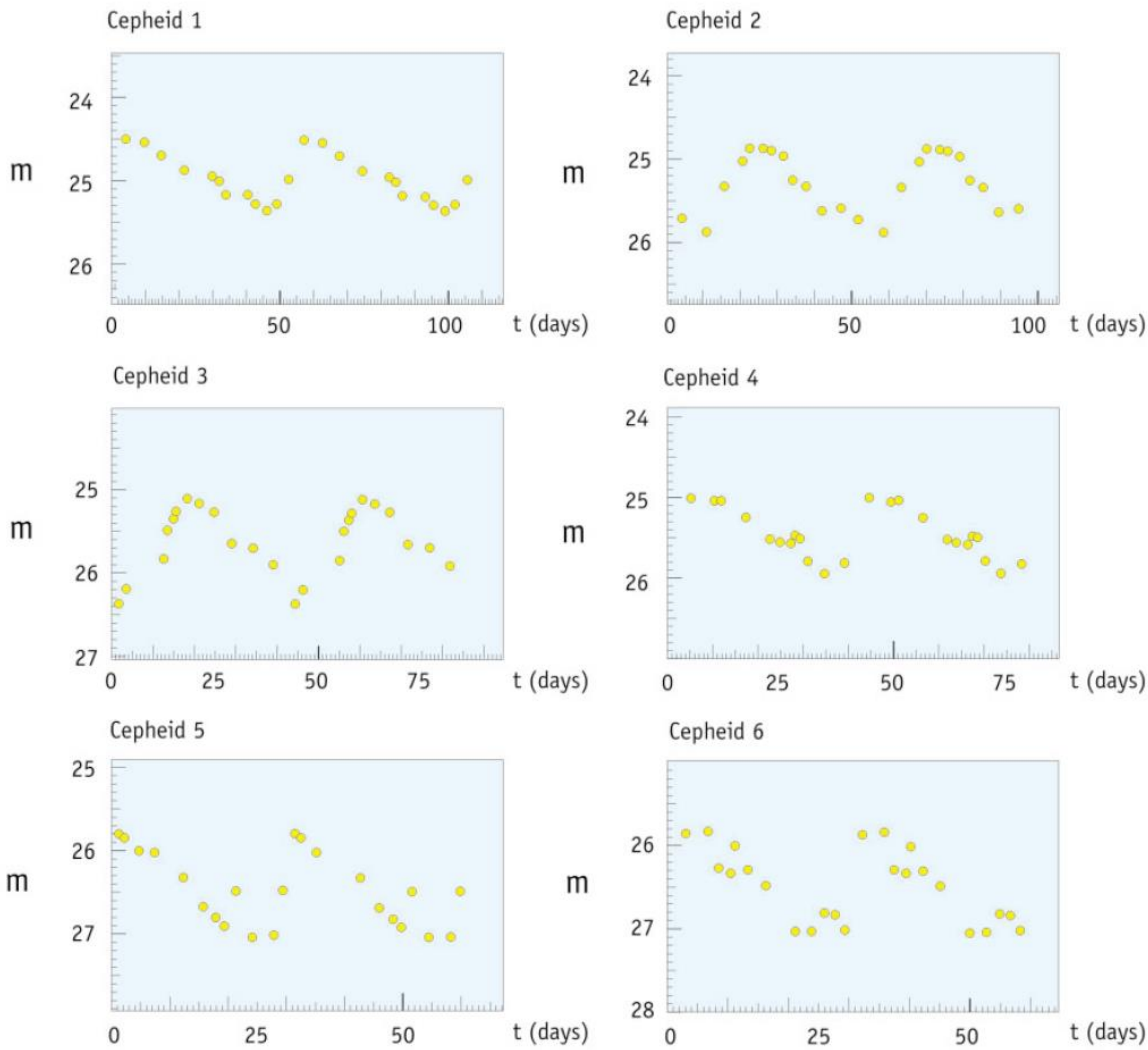
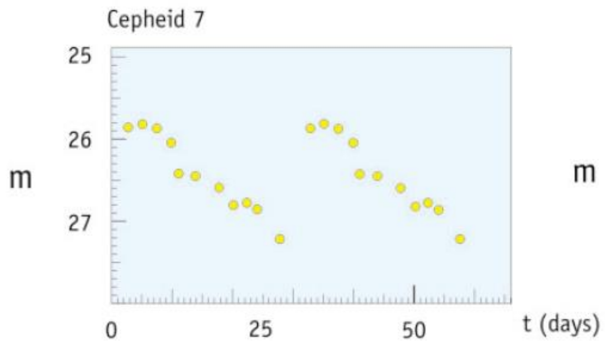


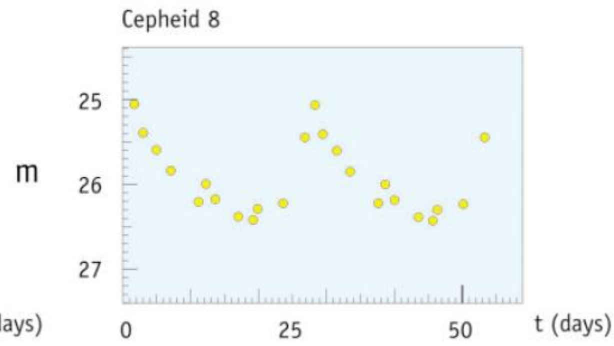
Figure 3. Three snapshots of a Cepheid variable in M100 from Hubble Space Telescope observations.

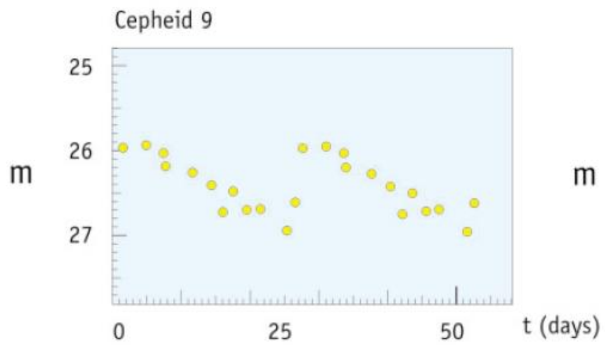
These observations were processed to produce the light curves of these stars, that is, dependence of apparent magnitude *m* on time. Below you will find the light curves of 12 selected Cepheids used in the article by Freeman et al. (1994).

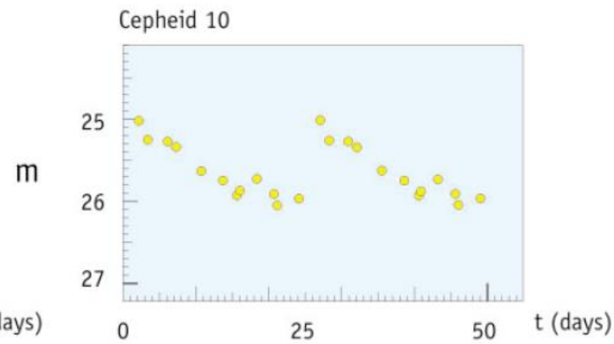
To find the distance to the M100 galaxy, you will first determine the apparent magnitude and pulsation periods of these Cepheids from the observation data. Then you will compute the absolute magnitudes of these stars from the period-luminosity relation. From this you will estimate the distance to each of the stars, critically assess and finally average the valid values to obtain the distance to the galaxy.

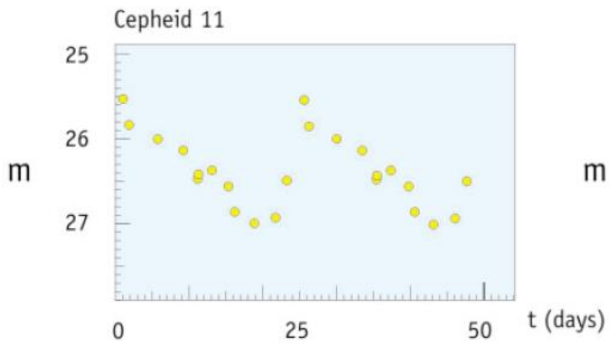


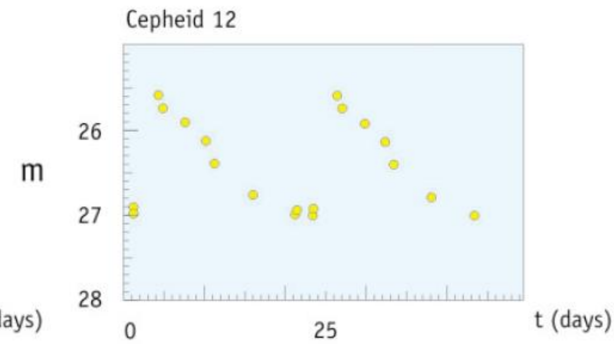












A For each of the stars, read the graphs to determine the pulsation period *P* (in days), the maximum m_{max} and minimum m_{min} apparent magnitude and compute the average magnitude m_{av} from these two values.

Answer: Fill in the table (1 p for each correct value). Try to read the magnitude data from the graph with precision of 0.05 magnitudes and the time with precision of 0.5 days.

Star	Period, days	Minimum magnitude	Maximum magnitude	Average magnitude
Cepheid 1				
Cepheid 2				
Cepheid 3				
Cepheid 4				
Cepheid 5				
Cepheid 6				
Cepheid 7				
Cepheid 8				
Cepheid 9				
Cepheid 10				

Star	Period, days	Minimum magnitude	Maximum magnitude	Average magnitude
Cepheid 11				
Cepheid 12				

Solution: to get the answer, the values should be read from graphs. Allow the student to mix up minimum and maximum magnitudes, since minimum value corresponds to maximum brightness. This should not affect the average value and the further solution.

Star	Period, days	Minimum magnitude	Maximum magnitude	Average magnitude
Cepheid 1	53.5	25.30	24.50	24.90
Cepheid 2	47.5	25.90	24.90	25.40
Cepheid 3	42.5	26.40	25.10	25.75
Cepheid 4	39.0	25.95	25.00	25.475
Cepheid 5	31.0	27.05	25.80	26.425
Cepheid 6	29.0	27.10	25.80	26.45
Cepheid 7	30.5	27.20	25.80	26.50
Cepheid 8	27.0	26.40	25.05	25.725
Cepheid 9	26.0	27.00	25.90	26.45
Cepheid 10	24.5	26.10	25.00	25.55
Cepheid 11	24.0	27.00	25.55	26.275
Cepheid 12	22.0	27.00	25.60	26.30

The current best estimate for the period-luminosity relation for Cepheids is:

$$M = -2.78 \log P - 1.35$$

where $\log P$ is the decimal logarithm of the pulsation period *P* measured in days, and *M* is the absolute magnitude of the star.

B Determine the absolute magnitude of each of the cepheids from the pulsation period.

Answer: Fill in the table below (1 p. for each correct value)

Cepheid	1	2	3	4	5	6	7	8	9	10	11	12
Absolute magnitude *M*												

Solution: Apply equation to the measured periods. For example, for Cepheid 1, the absolute magnitude is $M_1 = -2.78 \lg 53.5 - 1.35 = -2.78 \cdot 1.73 - 1.35 = -6.15^m$

Cepheid	1	2	3	4	5	6	7	8	9	10	11	12
Absolute magnitude *M*	-6.15	-6.01	-5.88	-5.77	-5.50	-5.42	-5.48	-5.33	-5.28	-5.21	-5.19	-5.08

C Determine now the distance to each Cepheid by comparing their apparent *m* and absolute *M* magnitudes. Neglecting the interstellar attenuation, these magnitudes are related by relation

$$M = m + 5\log d\ [\mathrm{pc}] - 5$$

where $\log d[\mathrm{pc}]$ stands for decimal logarithm of the distance expressed in parsecs. Its is more convenient to express the distance to galaxies in megaparsecs, Mpc, where 1 Mpc = 10^6 pc.

Answer: Fill in the table below. (1 p for each correct value)

Cepheid	1	2	3	4	5	6	7	8	9	10	11	12
Distance *d*, Mpc												

Solution: Express the decimal logarithm of distance as

$$\log d[\mathrm{pc}] = 0.2(M - m) + 1$$

Or, using that $\log d[\mathrm{pc}] = \log d[\mathrm{Mpc}] + 6$,

$$\log d[Mpc] = 0.2(M - m) - 5$$

For example, the distance to Cepheid 1 is computed from $\log d[\mathrm{Mpc}] = 0.2 \cdot (-6.15 - 24.90) - 5 = 0.2 \cdot (-31.05) - 5 = 6.21 - 5 = 1.21$. Then $d = 10^{1.21} = 16.22$ Mpc. The slightly different values in the Table below are taken directly from the source, so should be most precise.

Cepheid	1	2	3	4	5	6	7	8	9	10	11	12
Distance *d*, Mpc	16.25	19.15	21.15	17.77	24.22	23.61	24.85	16.25	22.22	14.20	19.61	18.90

Now look at your result. If one or two values in your results are strongly different from others, you may exclude them from further estimate computations. For example, if your computed distances are 12, 11, 546, 14, 13 Mpc, then the 546 Mpc result may safely be excluded as error of measurement or of calculation.

D If you did so, please write down the numbers of the excluded Cepheids.

Answer: I have decided to exclude the following Cepheids from the further computations, as the obtained distances are not consistent with others: [ ] (no points)

Solution: The main purpose is to help the students to exclude the obviously incorrect values. The only value that may look like it should be excluded from the real dataset is Cepheid 10, but the value is not an error but result of the measurement data uncertainties.

E After excluding the outliers, determine the distance to the M100 galaxy as the average of all valid distance values.

Answer: The distance to the M100 galaxy is [ ] Mpc (6 p.)

Solution: The average from all these distance values is 19.85 Mpc.

Using the full Cepheid measurement dataset and taking into account the interstellar absorption correction, the distance to the M100 galaxy was determined to be 17.1 Mpc with the uncertainty 1.8 Mpc (Freeman et al., 1994). Use that value in the next questions of the problem, and not the value you obtained.

F The cosmological recession velocity of the Virgo cluster, of which M100 is a member, is about 1200 km/s with the uncertainty of 100 km/s. Determine the Hubble constant from the distance value and the recession velocity. Determine also uncertainty of the Hubble constant from the uncertainty of the distance and of the recession speed.

Answer: From the observations of M100, the Hubble constant equals [ ] km/s/Mpc (2 p) with the uncertainty of [ ] km/s/Mpc. (5 p)

Solution: Using Hubble-Lemaitre's law, the Hubble constant is $H_0 = \frac{v}{d} = \frac{1200}{17.1} = 70.2$ km/s/Mpc (accepted values: from 70.0 to 71.0 km/s/Mpc). To estimate the uncertainty, let us compute the minimum and maximum Hubble constant values from values of the distance and speed within their respective uncertainties.

$$H_{0,max} = \frac{1200 + 100}{17.1 - 1.8} = \frac{1300}{15.3} = 85.0 \frac{\text{km}}{\text{s Mpc}}$$

$$H_{0,min} = \frac{1200 - 100}{17.1 + 1.8} = \frac{1100}{18.9} = 58.2 \frac{\text{km}}{\text{s Mpc}}$$

The half of difference of these values, 13 km/s/Mpc, is then the uncertainty of the Hubble constant. The result of the M100 observations is then $H_0 = 70 \pm 13$ km/s/Mpc.

There are multiple methods to determine uncertainties, and we accept uncertainty estimates from 8 to 15 km/s/Mpc as correct.

G The age of the Universe *t* may be roughly estimated as the inverse of the Hubble constant, $t = 1/H_0$. Compute the age of the Universe that corresponds to the Hubble constant of 72 km/s/Mpc (*this may differ from the value you obtained before*). Be careful when converting the units.

Answer: Approximate age of the Universe is [ ] billion years. (5 p.)

Solution:

Inserting the Hubble constant value into the age expression, we obtain

$$t = \frac{1}{H_0} = \frac{1}{72}\frac{\text{s Mpc}}{\text{km}} = \frac{1}{72} \cdot \frac{y}{3.1557 \cdot 10^7} \cdot \frac{3.086 \cdot 10^{22}\ \text{m}}{10^3\ \text{m}} = 1.358 \cdot 10^{10}\ y$$

This is 13.58 billion years (one billion is 1000 million).

General comments:

The problem is based on an exercise from the ESA/ESO Astronomy Exercise Series. The scientific publication analyzing the data is
Freedman, W.L. *et al.*, Nature, vol. **371**, pp. 757-762 (1994): *Distance to the Virgo cluster galaxy M100 from Hubble Space Telescope observations of Cepheids*.